2. Successive Wins

To encourage Elmer's promising tennis career, his father offers him a prize if he wins (at least) two tennis sets in a row in a three-set series to be played with his father and the club champion alternately: father-champion-father or champion-father-champion, according to Elmer's choice. The champion is a better player than Elmer's father. Which series should Elmer choose?

Solution for Successive Wins

Since the champion plays better than the father, it seems reasonable that fewer sets should be played with the champion. On the other hand, the middle set is the key one, because Elmer cannot have two wins in a row without winning the middle one. Let C stand for champion, F for father, and W and L for a win and a loss by Elmer. Let f be the probability of Elmer's winning any set from his father, c the corresponding probability of winning from the champion. The table shows the only possible prize-winning sequences together with their probabilities, given independence between sets, for the two choices.

	Father first				Champion first			
Set with:	F	C	F	Probability	C	F	C	Probability
	W	W	W	fcf	W	W	W	cfc
	W	W	L	$fc(1-f)$	W	W	L	$cf(1-c)$
	L	W	W	$(1-f)cf$	L	W	W	$(1-c)fc$
Totals				$fc(2-f)$				$fc(2-c)$

Since Elmer is more likely to best his father than to best the champion, f is larger than c, and $2 - f$ is smaller than $2 - c$, and so Elmer should choose CFC. For example, for $f = 0.8$, $c = 0.4$, the chance of winning the prize with FCF is 0.384, that for CFC is 0.512. Thus the importance of winning the middle game outweighs the disadvantage of playing the champion twice.

Many of us have a tendency to suppose that the higher the expected number of successes, the higher the probability of winning a prize, and often this supposition is useful. But occasionally a problem has special conditions that destroy this reasoning by analogy. In our problem the expected number of wins under CFC is $2c + f$, which is less than the expected number of wins for FCF, $2f + c$. In our example with $f = 0.8$ and $c = 0.4$, these means are 1.6 and 2.0 in that order. This opposition of answers gives the problem its flavor. The idea of independent events is explained in PWSA, pp. 81–84.